Financial Math

Simple interest:

$I = Prt$

$A = P + I$

I: Interest ($)
P: Money invested (principal) ($)
r: Annual interest rate (decimal)
t: time (years)
A: Amount ($) (at the end of period)

…….

Compound Interest:

$A = P(1+i)^n$

$$P = \frac{A}{(1+i)^n}$$

P: Money invested (principal) ($)
A: Amount (at the end of compounding period) ($)
i: interest rate for each compounding period (decimal)
n: number of compounding periods

Example:
Calculate the amount of $2000 for 5 years at 8% compounded quarterly.

$$i = \frac{0.08}{4} = 0.02$$

n = 4x5 = 20

$A = P(1+i)^n = 2000\ (1+0.02)^{20} = 2972$

……..

Annuities

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$P = \frac{R[1-(1+i)^{-n}]}{i}$$

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$R = \frac{Ai}{(1+i)^{n}-1}$$

R: Regular deposit made at the end of each period for n periods ($)
A: Amount of annuity at the end of n^{th} period) ($)
P: Present value of the annuity ($)

Example:

Calculate the amount of annuity for $200 deposited every 6 months by 6% for 3 years.

$$A = \frac{R[(1+i)^{n}-1]}{i} = \frac{200[(1+0.03)^{6}-1]}{0.03} = 1294$$

Mortgages

Mortgage interest rate is compounded semi-annually (6 months) by Canadian laws. But people want to make payment monthly.

Example:
Stephanie buys a house. She needs a mortgage of $ 200, 000. Mortgage rate: 8%, mortgage period: 20 years. Find the monthly payment.

i: monthly rate (decimal)
Mortgage rate (decimal) = 0.08
Number of payments: n

Semi-annual rate: $\frac{0.08}{2} = 0.04$

$(1+i)^6 = 1+0.04 = 1.04$

$i = 0.0065581969 \approx 0.0065582$

$n = 20x12 = 240$

$$P = \frac{R[1-(1+i)^{-n}]}{i}$$

solve for R

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{(200000)(0.0065581969)}{1-(1+0.0065581969)^{-240}} = 1656.72$$

The monthly payment is $1656.72