Arithmetic Sequences and Series

Arithmetic sequences: a, a+d, a+2d, a+3d,…,a+(n-1)d

Arithmetic series (sum of the terms of an arithmetic sequence):

a + (a+d) +(a+2d) + (a+3d) +…+ [a+(n-1)d]

$$t_n = a + (n-1)d$$

$$d = t_n - t_{n-1}$$

$$S_n = \left(\frac{a + t_n}{2} \right) \times n$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Geometric Sequences and Series

Geometric Sequences: a, ar^2, ar^3, …ar^{n-1}

Geometric Series (sum of the terms of an geometric sequence): a + ar^2 + ar^3 + …+ r^{n-1}

$$t_n = ar^{n-1}$$

$$r = \frac{t_n}{t_{n-1}}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad r \neq 1$$

a: First term (t_1)
t_n: nth term
n: Number of terms
S_n: Sum of the first n terms

d: common difference
r: Common ratio

Recursion Formulas:
In the recursion Formulas, the nth term is not calculated directly. Instead the first term is specified, and then next term will be calculated. Sequences and series of recursion formulas may be arithmetic, geometric or neither.

Recursive formula for the arithmetic sequence: $t_1 = a$ $\quad tn = tn\text{-}1 - d$ $\quad n \geq 2$

Recursive formula for the geometric sequence: $t_1 = a$ $\quad tn = r \times tn\text{-}1$ $\quad n \geq 2$

Fibonacci Sequence: It is neither arithmetic nor geometric

$t_1 = 1$ $\quad t_2 = 1$ $\quad t_n = t_{n-1} + t_{n-2}$ $\quad n>2$